MATHS AND STATS

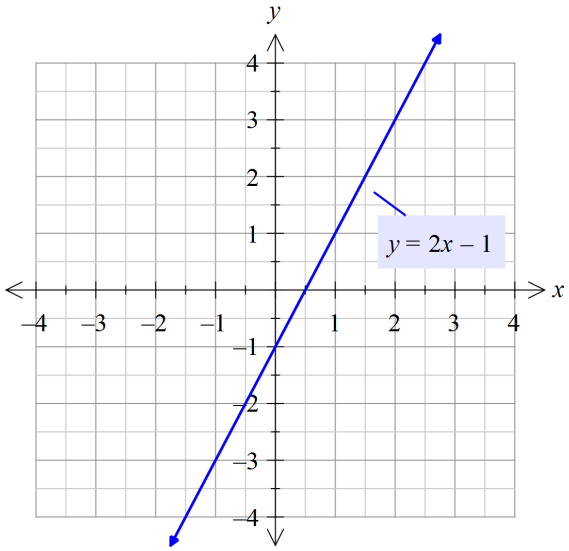
Graphing Linear Equations

Each point on a graph has co-ordinates (x, y) where x is the horizontal position of the point and y is the vertical position. A line on a graph is the set of all points which satisfy some mathematical rule, which is given by the line equation.

Eg For the line equation: $y = 2x - 1$

Point (0, –1) is on the line as –1 = 2 x (0) – 1 is true.
Both sides of = come to the same value
(Note that we put the -1 in place of y and the 0 in place of x in the equation.)

Point (-3, 2) is not on the line, as $2 \neq 2 \times (-3) - 1$



The same goes for (1, 1), or (10, 19), and also for, (0.1, –0.8), or (–2.25, –5.5).

Any point, which is on the line $y = 2x - 1$ will make the equation 'work'.
(i.e. when you substitute the values for x and y in the equation, it is true.)

Gradient

- The gradient or slope of a line is given by $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

The formula is used when you know two points (x_1, y_1) and (x_2, y_2) on your line.
Gradient can also be thought of as the rate of change in y as x changes: Every time you move 1 unit right, you move m units up.
A gradient is negative if it runs in the same direction as the sloping line in capital N.
Horizontal lines have gradient = 0, and vertical lines have infinite (∞) gradient.

- When an equation is written in the form, called Gradient – Intercept form

$$y = mx + b$$

m is the gradient and **b is the y-intercept** (as $y = 0x + b$ gives $y = b$)

x and y intercepts

To find where a line crosses the *x*-axis, set *y* to 0 and solve for *x*.
To find the *y*-intercepts, set *x* to 0 and solve for *y*.

Example

- Find the intercepts of $y = 0.5x + 2$.

When $y = 0$,

$$0 = 0.5x + 2$$
$$-2 = 0.5x$$
$$x = -4$$

x-intercept = -4 (-4, 0)

When $x = 0$,

$$y = 0.5 \times 0 + 2$$
$$y = 2$$

y-intercept = 2 (0, 2)

- Find the **gradient *m*** and **y intercept *b***
  $\boldsymbol{y = 0.5x + 2}$ is written in the form

  $$\boldsymbol{y = mx + b}$$

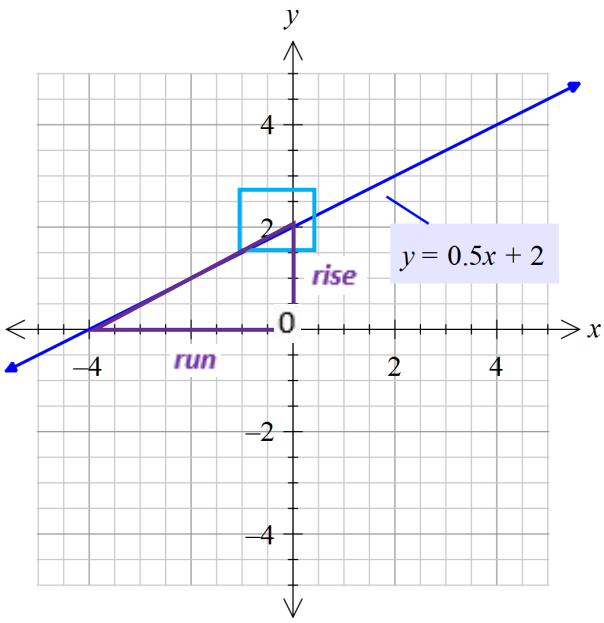
  so we get ***m = 0.5*** and ***b = 2***

- From the graph we can get the **gradient *m***,

  $$m = \frac{rise}{run} = \frac{2}{4} = 0.5$$

  (the slope is positive and we only want the lengths of the triangle sides shown)

  and ***y*-intercept *b* = 2**
  (where the line crosses the *y* axis



Other formulas

These can be used to find the equation of a line with the information specified

Equation from 2 points (x_1, y_1) and (x_2, y_2): $\dfrac{(y_2 - y_1)}{(x_2 - x_1)} = \dfrac{(y - y_1)}{(x - x_1)}$

Equation from point (x_1, y_1) and gradient *m*: $(y - y_1) = m(x - x_1)$

Gradient of a perpendicular line: $m_2 = -\dfrac{1}{m_1}$ where m_1 is the gradient of the other line.

Example

Find the equation of a line perpendicular to $2x - y = 4$ and passing through the point $(-4,5)$.

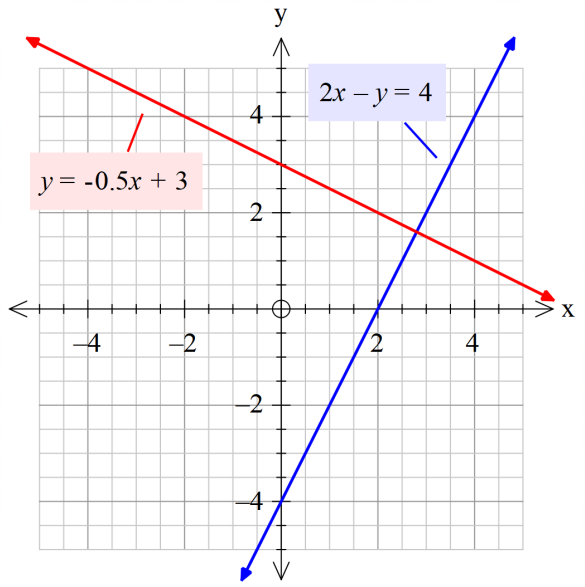
First we need the gradient of $2x - y = 4$

Rearrange to make *y* the subject: $y = 2x - 4$, $m_1 = 2$.

The gradient of the line we want is $m_2 = -\dfrac{1}{m_1}$ $m_2 = -\frac{1}{2}$.

Using the point and gradient formula with $m = -\frac{1}{2}$ and point $(-4,5)$

$$(y - y_1) = m(x - x_1)$$
$$y - 5 = -\frac{1}{2}(x + 4)$$
$$y - 5 = -\frac{1}{2}x - 2$$
$$y = -\frac{1}{2}x + 3$$



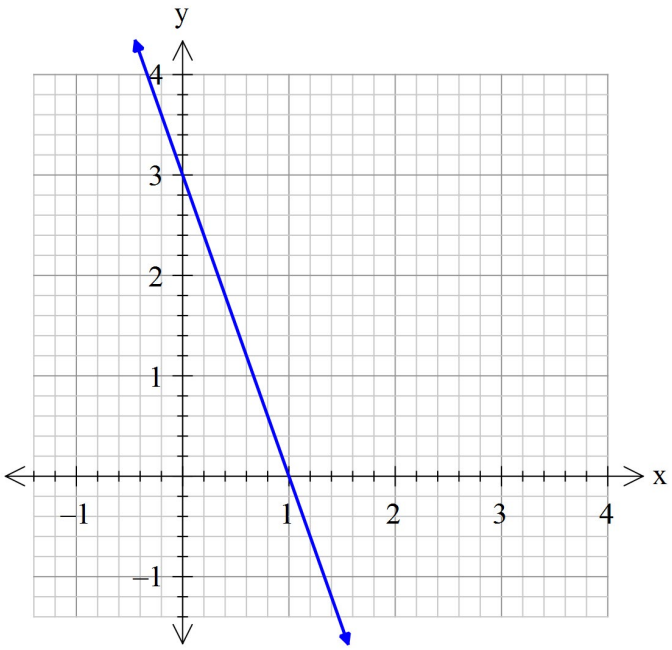
Exercises

1. For the line $y = 4x - 7$ find the
   (i) gradient
   (ii) y-intercept
   (iii) x-intercept

2. Find the equation of the line through the points (1, 2) and (5, 8)

3. Is (2, 5) on the line $y = 3x + 2$

4. For this graph (next page), find the
   (i) gradient
   (ii) y-intercept
   (iii) equation of the line



5. Write the equation of a line parallel to $y = -4x + 5$

6. Write the equation of a line perpendicular to $y = 3x - 9$

7. What point on the line $y = -\frac{2}{3}x - 7$ has an *x*-coordinate of 9?

8. Find the equation of the line parallel to $2y - 6x - 3 = 0$ passing through the point (1, -2).

Answers

1. (i) $m = 4$
   (ii) -7
   (iii) $\frac{7}{4}$
2. $y = \frac{3}{2}x + \frac{1}{2}$
3. No
4. (i) $m = -3$
   (ii) 3
   (iii) $y = -3x + 3$
5. Many answers all of the form $y = -4x + b$ eg $y = -4x + 3$
6. Many answers all of the form $y = -\frac{1}{3}x + b$ eg $y = -\frac{1}{3}x + 2$
7. (9, -13)
8. $y = -\frac{1}{3}x - \frac{5}{3}$