Solving equations involving logs and exponentials

Solving basic log and exponential equations

- If the equation is in index form, change it to log form to solve for the unknown.

Find x when $\qquad 10^x = 25 \qquad \longleftrightarrow \qquad \log_{10} 25 = x$

Using the calculator $x \approx 1.398$

- If the equation is in log form, change it to index form to solve for the unknown.

Find x when $\qquad \log_{10} x = 1.9 \qquad \longleftrightarrow \qquad 10^{1.9} = x$

Using the calculator $x \approx 79.433$

Exercises A
Change from log to index form or vice-versa to find the value of x.

Note: $\log_e$ is denoted $\ln$ on the calculator and $\log_{10}$ is denoted $\log$ on the calculator.

a) $\log_{10} x = 0.5$

b) $\log_{10} x = 1.2$

c) $10^x = 28$

d) $10^x = 45$

e) $e^x = 212$

f) $e^x = 96$

g) $\log_e x = 0.2$

h) $\log_e x = 1.8$

Solving log equations

Example

1. Solve $\quad \log_{10}(10x - 5) = 2.$
$\log_{10}(10x - 5) = 2$ in index form is $10^2 = 10x - 5.$
So solve $100 = 10x - 5$
$105 = 10x$
$x = 10.5$

Exercises B

Solve the following equations for x

1) $\log_5(x) = 2$

2) $\log_3(20x) = 3$

3) $\log_{10}(4x) = 2$

4) $\log_4\left(\frac{x}{2}\right) = 6$

5) $\log_2\left(\frac{2x}{5}\right) = 1$

6) $\log_3\left(\frac{3x}{4} + 1\right) = 8$

Solving exponential equations

Example

Solve $5000 = 1500(1.08)^n$.

$$\frac{5000}{1500} = 1.08^n \qquad \text{(dividing both sides by 1500)}$$

$$\log\left(\frac{10}{3}\right) = \log(1.08^n) \qquad \text{(take log of both sides)}$$

$$\log\left(\frac{10}{3}\right) = n\log(1.08) \qquad \text{(using log rule C)}$$

$$\frac{\log\left(\frac{10}{3}\right)}{\log(1.08)} = n \qquad \text{(dividing both sides by } \log(1.08))$$

$$n \approx 15.6$$

Exercises C

1) Solve $50 = 5^n$

2) Solve $1000 = 100(1.02)^n$

3) Solve $6000 = 200(1.03)^n$

4) Solve $2500 = 500(1.005)^n$

5) Solve $1200 = 60(1.005)^{n/12}$

6) Solve $4500 = 2000(1.004)^{n/52}$

7) Solve $1500 = 1000(1.01)^{n/4}$

8) Solve $4000 = 4000(1.05)^{n/6}$

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Answers

Answers A

a) $\log_{10} x = 0.5 \rightarrow x = 10^{0.5} \rightarrow x = 3.16$

b) $\log_{10} x = 1.2 \rightarrow x = 10^{1.2} \rightarrow x = 15.85$

c) $10^x = 28 \rightarrow x = \log_{10} 28 \rightarrow x = 1.45$

d) $10^x = 45 \rightarrow x = \log_{10} 45 \rightarrow x = 1.65$

e) $e^x = 212 \rightarrow x = \log_e 212 \rightarrow x = 5.36$

f) $e^x = 96 \rightarrow x = \log_e 96 \rightarrow x = 4.56$

g) $\log_e x = 0.2 \rightarrow x = e^{0.2} \rightarrow x = 1.22$

h) $\log_e x = 1.8 \rightarrow x = e^{1.8} \rightarrow x = 6.05$

Answers B

1) $x = 25$

2) $x = \frac{27}{20}$

3) $x = 25$

4) $x = 8192$

5) $x = 5$

6) $x = 8746\frac{2}{3}$

Answers C

1) $n \approx 2.43$

2) $n \approx 116.28$

3) $n \approx 115.07$

4) $n \approx 322.69$

5) $n \approx 7207.72$

6) $n \approx 10563.16$

7) $n \approx 163.00$

8) $n = 0$