MATHS AND STATS

Operations with Surds

Numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and $\sqrt{6}$ are all irrational and are called surds.
To work efficiently with surds you need to know your perfect square numbers.

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256…

Simplifying surds

Rules: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\qquad$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples:

1.) $\frac{\sqrt{36}}{\sqrt{9}} = \sqrt{\frac{36}{9}} = \sqrt{4} = 2$

2.) $\sqrt{60} = \sqrt{4 \times 15} = \sqrt{4} \times \sqrt{15} = 2\sqrt{15}$

To simplify we look for factors that are squares. If you can't find the largest square factor you can always work in small steps.

Examples:

3.) $\sqrt{180} = \sqrt{9} \times \sqrt{20} = 3\sqrt{20} = 3\sqrt{4} \times \sqrt{5} = 3 \times 2 \times \sqrt{5} = 6\sqrt{5}$

4.) $3\sqrt{6} \times 2\sqrt{12} = 6\sqrt{72} = 6\sqrt{9 \times 8} = 6 \times 3 \times \sqrt{8} = 18\sqrt{4 \times 2} = 18 \times 2\sqrt{2} = 36\sqrt{2}$

Addition and subtraction

You can only add or subtract surds if they are the same type

Examples:

1.) $\sqrt{3} + 5\sqrt{3} - 2\sqrt{3} = 4\sqrt{3}$

2.) $5\sqrt{3} + 2\sqrt{7} - 3\sqrt{3} + 4\sqrt{7} = 2\sqrt{3} + 6\sqrt{7}$

More complex cases of simplifying surds

Sometimes you will need to use more than one process in order to simplify surds.

Example

1.)

$$\begin{aligned}\sqrt{8}-\sqrt{18}+\sqrt{50} &= \sqrt{4}\times\sqrt{2}-\sqrt{9}\times\sqrt{2}+\sqrt{25}\times\sqrt{2}\\ &= 2\sqrt{2}-3\sqrt{2}+5\sqrt{2}\\ &= 4\sqrt{2}\end{aligned}$$

Rationalising the denominator

To write an expression in simplest form we sometimes want to have a whole number as the denominator

Example:

1.) $\frac{\sqrt{6}}{\sqrt{5}}=\frac{\sqrt{6}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{30}}{5}$

Here we simply multiply by the denominator over itself, as it is a single surd.

More complex cases of rationalising the denominator

When the denominator is not just a single surd but something like $4+\sqrt{3}$ or $2-\sqrt{5}$ we can't simply multiply by the denominator as this will still leave us with an irrational denominator. We need to multiply by the conjugate of the denominator. If the denominator is $4+\sqrt{3}$, then the conjugate is $4-\sqrt{3}$ (just change the sign of the surd part)

Example:

2) Write $\frac{3}{2-\sqrt{7}}$ with a rational denominator

$$\frac{3}{2-\sqrt{7}} = \frac{3}{2-\sqrt{7}} \times \frac{2+\sqrt{7}}{2+\sqrt{7}}$$
$$= \frac{3(2+\sqrt{7})}{(2-\sqrt{7})(2+\sqrt{7})}$$
$$= \frac{6+3\sqrt{7}}{4+2\sqrt{7}-2\sqrt{7}-\sqrt{7}\times\sqrt{7}}$$
$$= \frac{6+3\sqrt{7}}{4-7}$$
$$= \frac{6+3\sqrt{7}}{-3} \quad or \quad -2-\sqrt{7}$$

3.) Rationalize the denominator of $\frac{2+\sqrt{3}}{3+\sqrt{5}}$

$$\frac{2+\sqrt{3}}{3+\sqrt{5}} = \frac{2+\sqrt{3}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$$
$$= \frac{(2+\sqrt{3})\times(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$
$$= \frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{3}\times\sqrt{5}}{9+3\sqrt{5}-3\sqrt{5}-\sqrt{5}\times\sqrt{5}}$$
$$= \frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{9-5}$$
$$= \frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{4}$$

This cannot be simplified any more.

Exercises

1.) Simplify the following surds

(a) $\sqrt{80}$

(b) $3\sqrt{54}$

(c) $\sqrt{\frac{25}{4}}$

(d) $5\sqrt{128}$

2.) Simplify the following surds

(a) $5\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$

(b) $\sqrt{150} - 2\sqrt{2} + 4\sqrt{6}$

3.) Write the following in simplest form

(a) $\sqrt{6} \times \sqrt{10}$ (b) $\sqrt{5} \times \sqrt{7}$ (c) $5\sqrt{12} \times 3\sqrt{2}$

4.) Write the following with a rational denominator

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{\sqrt{10}}{2\sqrt{5}}$

(d) $\frac{4}{3-\sqrt{2}}$

(e) $\frac{2}{3-\sqrt{5}}$

(f) $\frac{3\sqrt{3}}{5+\sqrt{2}}$

Answers

1. (a) $4\sqrt{5}$
   (b) $9\sqrt{5}$
   (c) $\frac{5}{2}$
   (d) $8\sqrt{2}$

2. (a) $7\sqrt{7}$
   (b) $9\sqrt{6} - 2\sqrt{2}$

3. (a) $2\sqrt{15}$
   (b) $\sqrt{35}$
   (c) $30\sqrt{6}$

4. (a) $\frac{\sqrt{5}}{5}$
   (b) $\frac{2\sqrt{3}}{3}$
   (c) $\frac{\sqrt{50}}{10}$ or $\frac{\sqrt{2}}{2}$
   (d) $\frac{12+4\sqrt{2}}{7}$
   (e) $\frac{3+\sqrt{5}}{2}$
   (f) $2\sqrt{3} - \sqrt{6}$