Expanding and Factorising

Sometimes algebraic expressions have brackets in them. We need to know how to write them without brackets. This is called 'expanding' the expression.

Examples:

$$4(3x + 2) = 12x + 8$$

There is an invisible multiplication sign between the 4 and the brackets. Note that both terms inside the brackets are multiplied by the 4.
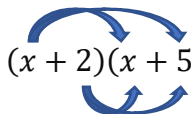
$$2x(x^2 + 5x - 6) = 2x^3 + 10x^2 - 12x$$

All terms inside the brackets are multiplied by the outside term

Multiplying Binomials

What if there are two sets of brackets? Again, the multiplication sign is invisible: $(x + 2)(x + 5)$

Both of the terms in the first set of brackets must be multiplied by both terms in the second set of brackets. i.e., there are 4 multiplications to do. A common way to remember this is FOIL: First, Outside, Inside, Last

$$(x + 2)(x + 5) = x^2 + 5x + 2x + 10$$
$$= x^2 + 7x + 10$$

Factorising (common factor)

Factorising is the reverse process. We need to write the expressions as the product of two or more terms. To factorise algebraic expressions, look for the highest common factor (of all terms). This common factor is brought out the front of the brackets, and remaining factors (after dividing the common factor out) stay inside the brackets.

$$4x + 8y = 4(x + 2y)$$

$$16x^2 + 8x = 8x\,(2x + 1)$$

Common factor of these terms is 4
So dividing each term by 4 leaves $x + 2y$

Here it is $8x$
$16x^2$ divided by $8x$ is $2x$

If the terms are complicated, consider the numbers first, then each letter:

$$30x^2y^2 + 15xy^2 - 25y^3$$

$$= 5y^2(6x^2 + 3x - 5y)$$

5 is the common factor of the numbers in the 3 terms
There is no common x since not every term has an x
For y, the common factor is y^2
So the common factor is $5y^2$

You can always check your factorisation by expanding your answer to see if you get back where you started!

Expanding and Factorising Exercises

1) Expand the following:
   a) $7(3x + 5)$
   b) $(x + 5)(x + 8)$
   c) $(4x + 7)(5a + 3)$
   d) $3x(3x^3 + x^2 + 4)$
   e) $(x + 5)(x - 5)$
   f) $(y - 4)(6y + 8)$
   g) $(x - 7)(x - 9)$

2) Factorise the following:
   a) $x^2 + 5x$
   b) $49a^2 - 49$
   c) $18xy + 24x$
   d) $16xy - 27xz$
   e) $7x^2y + 28x$
   f) $5x^2 - 15x + 45$
   g) $24x^2y^2 + 16x^2y + 8xy^2$
   h) $15ab^2 - 27ab + 12a^3 - 18ab^3$

Answers

1) Expanding
   a) $21x + 35$
   b) $x^2 + 13x + 40$
   c) $20ax + 12x + 35a + 21$
   d) $9x^4 + 3x^3 + 12x$
   e) $x^2 - 25$
   f) $6y^2 - 16y$
   g) $x^2 - 16x + 63$

2) Factorising
   a) $x(x + 5)$
   b) $49(a^2 - 1) = 49(a + 1)(a - 1)$
   c) $6x(3y + 4)$
   d) $x(16y - 27z)$
   e) $7x(xy + 4)$
   f) $5(x^2 - 3x + 9)$
   g) $8xy(3xy + 2x + y)$
   h) $3a(5b^2 - 9b + 4a^2 - 6b^3)$

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