

A general method for defining the number of cycles of repeated loading

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SUMMARY

In geotechnical engineering designs and safety checks, the number of cycles of repeated loading N is frequently used as a parameter for calculating the response of soil structures under the repeated loading. However, there appears to be a lack of rational methods for calculating N for repeated loadings along general stress paths. A new method for defining N is proposed in this paper, based on the assumption that a yield surface may be identified for the material. It is demonstrated that the proposed method provides a rational and useful means for engineering calculation of soil deformations under repeated loadings. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: cyclic loading; number of cycles; pore pressure development

1. INTRODUCTION

The foundations of some engineering structures are frequently subject to repeated loadings involving potentially large numbers of cycles. Some examples of these loadings are storm effects on offshore structures, seismic ground movements during an earthquake, and the effects of traffic movement on pavements. The consequences of repeated loading vary widely, from improving the strength and stiffness of the soil to inducing catastrophic failure. In geotechnical engineering practice, perhaps the most important analytical approach for considering the effects of such repeated loading has been to express the response of the geomaterial in terms of N , the number of cycles of loading, via empirical formulae (e.g. References [1–5]). However, it appears that there is no consistent definition of N , which is both rational and suitable for general loadings, and in particular is applicable where the magnitude of the cyclic loading varies. Consequently, a key parameter in calculating the effect of repeated loading, the value of N , is usually estimated arbitrarily if the stress path is not identical for each cycle [6–8,4]. In this article, a general and rational method for calculating N for repeated loadings is proposed. It has the advantage that existing empirical equations and some constitutive models can be applied to repeated loading along general stress paths. Examination of the applicability of the proposed method for some practical cases is made.

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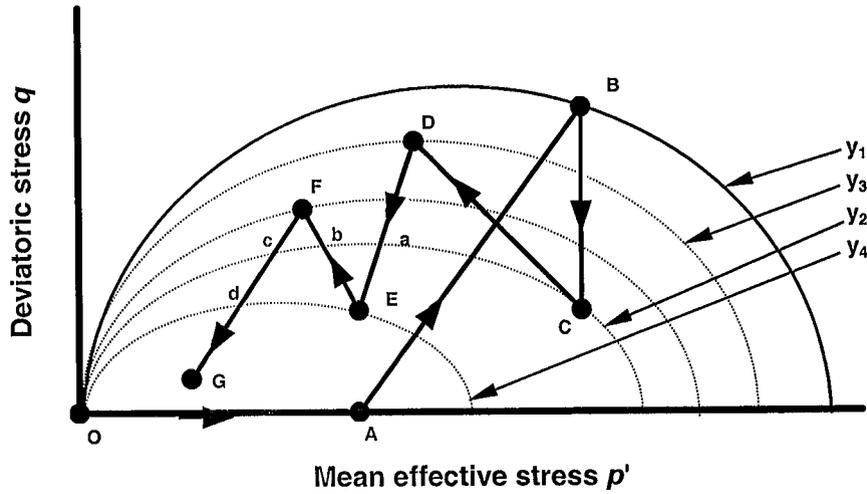


Figure 3. Loading along stress path OABCDEF.

- (2) Cases where the stress path for the repeated loading is not the same for each cycle, i.e. non-uniform repeated loadings (Figure 3).

In order to determine the cycle number for repeated loadings for more complicated stress paths, a general method is required. In the method proposed here, the cycle number, N , is associated with the variation of a yield surface, not necessarily with the variation of a stress state. The former describes events in at least a two-dimensional space (e.g. the change in a characteristic area in $p' - q$ space), while the latter describes events in one-dimensional space (e.g. the change in a characteristic length of the stress path).

For the purpose of illustration, in this paper the modified Cam clay model [9] is examined. However, the method for determining N is quite general and the method could easily be extended to other yield surfaces. The modified Cam clay yield surface is elliptical and may be expressed as

$$f = \left(\frac{q}{0.5Mp'_0}\right)^2 + \left(\frac{p' - 0.5p'_0}{0.5p'_0}\right)^2 - 1 = 0 \tag{1}$$

where the mean stress p' is given by

$$p' = \frac{1}{3}(\sigma'_{11} + \sigma'_{22} + \sigma'_{33}) \tag{2}$$

and the generalized deviatoric stress q by

$$q = \frac{1}{\sqrt{2}} \sqrt{[(\sigma'_{11} - \sigma'_{22})^2 + (\sigma'_{22} - \sigma'_{33})^2 + (\sigma'_{33} - \sigma'_{11})^2 + 6(\sigma'^2_{12} + \sigma'^2_{23} + \sigma'^2_{31})]} \tag{3}$$

M is the stress ratio (q/p') at critical state, and p'_0 is the size of the current yield surface. σ'_{ij} represents the Cartesian components of the effective stress state.

For a reconstituted soil, the current yield surface is the maximum yield surface ever associated with that soil. Virgin loading involves stress increments which go outside the original yield surface

and cause it to expand. A new surface, described as a pseudo-yield surface, is introduced to describe stress variation inside the current yield surface. This is a surface in stress space on which the current stress state always remains. The pseudo-yield surface has a shape similar to the virgin yield surface, e.g. if the yield surface is elliptical then the pseudo-yield surface will also be an ellipse with the same aspect ratio. The size of the pseudo-yield surface is denoted by p'_c , its interception on the p' -axis.

The value of N for repeated loading is related to stress variations inside the current yield surface, and thus it is associated with the variation of the pseudo-yield surface. It is suggested that N should be set equal to 1 for the first retreat inside the yield surface following each episode of virgin yielding. A stress increment which causes the pseudo-yield surface to expand is defined as loading; one that causes it to shrink is unloading. An example is given below to illustrate the proposed method for calculating the number N .

Suppose a reconstituted soil is loaded for the first time from the origin O, along the stress path OABCDEFG (Figure 3). The variation of the number of cycles N for loading along OABCDEFG is calculated as follows. For stress path OAB, the yield surface expands monotonically, and the stress path involves virgin loadings. Stress path BCDEFG is inside the yield surface created originally by stress path OAB, and includes subsequent loading or reloading. For stress path BC, the pseudo-yield surface shrinks. Hence, stress path BC is unloading, with unloading first occurring at B, where the pseudo-yield surface starts to shrink. The pseudo-yield surface shrinks for the first time along BC, and therefore, this section of stress path describes unloading for the first time, i.e. $N = 1$. Reloading commences at C and for stress path CD the pseudo-yield surface expands for the first time but remains inside the current yield surface, i.e. $N = 1$.

Stress path DE is unloading, and $N = 2$ over that part of the stress path described by the interval Da, while $N = 1$ for the part of stress path described by the interval aE. The reason for this distinction is as follows. For unloading along BC, the pseudo-yield surface decreases from y_1 to y_2 , and furthermore, the reduction of the pseudo-yield surface associated with stress path Da is included for the first time. During unloading along DE, the pseudo-yield surface decreased from y_3 to y_4 , and the reduction in the pseudo-yield surface associated with stress path Da involves unloading for the second time, i.e. $N = 2$. However, stress path aE involves unloading for the first time, and therefore, $N = 1$. Similarly, stress path Eb involves reloading with $N = 1$, and stress path bF is reloading with $N = 2$. Stress path Fc is unloading with $N = 3$, stress path cd is unloading with $N = 2$, and finally stress path dG is unloading with $N = 1$.

An alternative way to think of this procedure is as follows. For unloading, when the pseudo-yield surface shrinks from y_n to y_{n+1} , the area between pseudo-yield surface y_n and pseudo-yield surface y_{n+1} is traversed once. Suppose an unloading stress increment with its pseudo-yield surface shrinks from y to $y + \delta y$. The cycle number N for this unloading stress increment is equal to the total number of traverses of that area for the particular unloading between pseudo-yield surfaces y and $y + \delta y$. The value of N for reloading can be calculated in a similar way.

The following two points should be noted. Because the size of the yield surface increases with both the mean stress level and the deviatoric stress level, the cycle number N is dependent on stress level in two aspects: the determination of the current yield surface and the variation in the area of the pseudo-yield surface. The value of N is calculated for stress excursions inside the current yield surface. This surface is defined by the stress level a soil has previously experienced. The value of N is set equal to 1 for the first retreat inside the yield surface following each episode of virgin yielding. Compared with the traditional methods, i.e. those in which a non-uniform repeated loading is represented by an equivalent uniform repeated loading with an equivalent

number of cycles at a reference stress level (e.g. Reference [8]) the proposed method makes it possible to analyse the effects of the order of cycles of non-uniform repeated loading.

3. APPLICATIONS

3.1. Pore pressure development

In checks for the possibility of liquefaction of an offshore foundation under cyclic loading caused by storms or earthquakes, the empirical equation proposed by De Alba *et al.* [10] is often used to compute the pore pressure development under repeated undrained loading [8, 11]. This equation is given as

$$\frac{u}{\sigma'_{v,0}} = \frac{2}{\pi} \arcsin\left(\frac{N}{N_1}\right)^{1/2\theta} \quad (4)$$

where N is the number of cycles, u is the value of the excess pore pressure at cycle N , $\sigma'_{v,0}$ is the initial vertical effective stress, N_1 is the number of cycles required to induce liquefaction in the soil at a given initial stress state $\sigma'_{v,0}$ and a given total stress path, and θ is a material parameter. Equation (4) is an empirical formula based on data from tests which have the following features: (1) the test is an undrained test, and (2) the total stress path for each cycle of the test is the same. Consequently, the equation is applicable to tests in which the features are the same as or similar to those described above. However, during earthquake loading or storm loading, the cyclic loading is most usually made up of cycles of different magnitudes. For example, the make-up of a typical design storm is as shown in Table I (after Bjerrum [6]). It appears that until now there has been no rational method for applying equation (4) to loading situations with non-uniform repeated loading, because there has been no consistent method to define the cycle number N for use with Equation (4).

Based on the proposed definition of the number of cycles presented above, Equation (4) can now be applied to describe the pore pressure development under non-uniform cyclic loading. The loading applied by the design storm listed in Table I is examined as an example.

In the engineering computation of the development of pore pressure for an offshore foundation, a widely used method assumes that the total stress path applied to the foundation by a wave is determined by the wave height, irrespective of the sequence of its occurrence [11]. Consequently,

Table I. Typical characteristics of a storm assumed in design (after Bjerrum [6]).

Wave group	Wave height (m)	Wave period (s)	No. of waves in a storm
1	0.6	5	497
2	2.1	7.2	490
3	6.1	10	485
4	10.1	11.5	471
5	14.1	12.5	282
6	18	13.2	121
7	22	13.4	32
8	25	13.5	3

the total stress increment for the loadings applied by waves of the same height, or the same group, is the same. Therefore, the effect of the typical storm listed in Table I may be substituted as a series of repeated loadings with eight groups of uniform repeated loadings. It is highly likely, therefore, that the order of the occurrence of waves of different heights will influence the total pore pressure produced.

To obtain the pore pressure increment due to the m th wave of the n th group of uniform repeated loading, Equation (4) is differentiated, to give

$$\frac{\delta u}{\sigma'_{v,0}} = \frac{\delta N}{\pi \theta N_{1,n} \sin^{2\theta-1}(u\pi/2\sigma'_{v,0}) \cos(u\pi/2\sigma'_{v,0})} \quad (5)$$

It is noted that the initial vertical effective stress, $\sigma'_{v,0}$, is a constant, that θ is a material constant and that only u varies with N . As illustrated in the previous section, the value of N is dependent on stress history, and N may take different values for any given linear reloading or unloading stress path. The group of waves applying uniform repeated loading is divided in such a way that the cycle number N increases in successive order. Suppose the value of the cycle number for the first wave of the n th group is $N(n,0) + 1$; then the value of N for the m th wave of the n th group is $N(n,0) + m$. Parameter $N_{1,n}$ is the number of cycles required to induce liquefaction at the given initial stress $\sigma'_{v,0}$ and under the same total stress path as imposed by the n th group of waves.

Integrating equation (5) from the end of the $(n-1)$ th group of cycles to the end of the m th cycle of loading in the n th group, the following equation is obtained:

$$\frac{\delta u}{\sigma'_{v,0}} = \frac{u|_{n-1}}{\sigma'_{v,0}} + \frac{1}{\pi \theta} \int_{N(n,0)}^{N(n,0)+m} \frac{\delta N}{N_{1,n} \sin^{2\theta-1}(u\pi/2\sigma'_{v,0}) \cos(u\pi/2\sigma'_{v,0})} - \frac{\Delta u_d}{\sigma'_{v,0}} \quad (6)$$

where $u|_{n-1}$ is the pore pressure at the end of the $(n-1)$ th group of uniform cycles, and Δu_d is included to account for any dissipation of pore pressure that may occur from the end of the $(n-1)$ th group of repeated loading to the end of the m th cycle of the n th group.

Equation (6) is applicable for repeated loadings along any stress path. Because N may take different values for one reloading or unloading stress path, it may be necessary to divide a stress path into several parts for integration. $N_{1,n}$ is dependent on the total stress path and its value is found from experimental data. Consequently, if a stress path is divided into several parts for integration, the values of $N_{1,n}$ are different for different parts.

If the value of N is the same for a complete reloading and unloading stress path in the n th group of repeated loading, a simplified expression for Equation (6) can be derived. Under this situation, $N_{1,n}$ is a constant, and N varies from $N(n,0)$ to $N(n,0) + m$. Based on the original Equation (4), the second term on the right-hand side of Equation (6) can be written as

$$\begin{aligned} & \frac{1}{\pi \theta N_{1,n}} \int_{N(n,0)}^{N(n,0)+m} \frac{\delta N}{\sin^{2\theta-1}(u\pi/2\sigma'_{v,0}) \cos(u\pi/2\sigma'_{v,0})} \\ &= \frac{2}{\pi} \left[\arcsin\left(\frac{N(n,0) + m}{N_{1,n}}\right)^{1/2\theta} - \arcsin\left(\frac{N(n,0)}{N_{1,n}}\right)^{1/2\theta} \right] \end{aligned} \quad (7)$$

A simplified equation for the pore pressure development is thus obtained

$$\frac{u}{\sigma'_{v,0}} = \frac{u|_{n-1}}{\sigma'_{v,0}} + \frac{2}{\pi} \left[\arcsin\left(\frac{N(n,0) + m}{N_{1,n}}\right)^{1/2\theta} - \arcsin\left(\frac{N(n,0)}{N_{1,n}}\right)^{1/2\theta} \right] - \frac{\Delta u_d}{\sigma'_{v,0}} \quad (8)$$

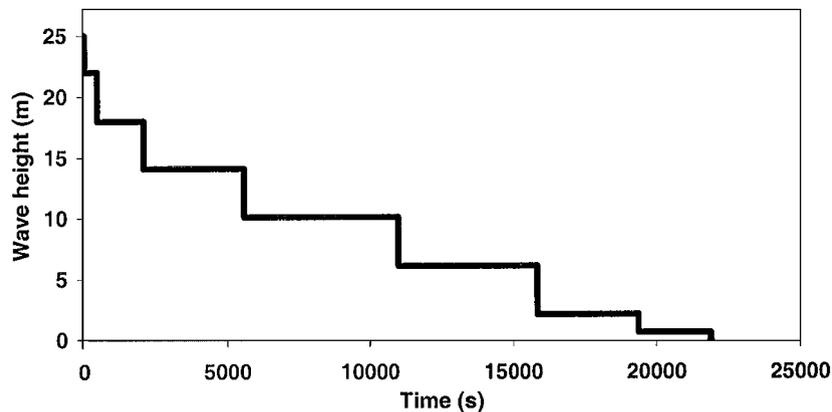


Figure 4. Sequence for the design waves (series 1).

Table II. Calculation of the number of cycles for Series 1.

Wave order	Wave height (m)	No. of waves in a storm	Value of N
1	25	3	From 1 to 3.
2	22	32	From 4 to 35.
3	18	121	From 36 to 156.
4	14.1	282	From 157 to 438.
5	10.1	471	From 439 to 909.
6	6.1	485	From 910 to 1394.
7	2.1	490	From 1395 to 1884.
8	0.6	497	From 1885 to 2381.

It is possible to consider the storm described in Table I as consisting of various sequences of parcels of the nominated waves. Two different sequences of the waves are considered here.

One possible sequence of the waves for the design storm is illustrated in Figure 4. Although such a sequence is highly unlikely to occur in practice, it does serve as a useful illustration of the proposed method for calculating N . In this sequence the biggest waves come first, followed by the smaller waves in parcels of descending wave height. Suppose the stress increment caused by the first wave is included inside the existing yield surface of the soil (i.e. virgin yielding will not occur for the loading caused by the entire group of waves) and the value of N for the first wave is equal to 1. The cycle numbers calculated according to the proposed method are then as listed in Table II.

For this series, the simplified Equation (8) can be employed directly to compute the development of pore pressure, because the value of N is the same for a complete reloading and unloading stress path in any one parcel of waves.

An alternative arrangement of the waves of the design storm is illustrated in Figure 5. As shown in the figure, all seven parcels of waves are divided into two sub-groups, except for the parcel with the biggest waves. Therefore, there are 15 parcels of uniform cyclic loading.

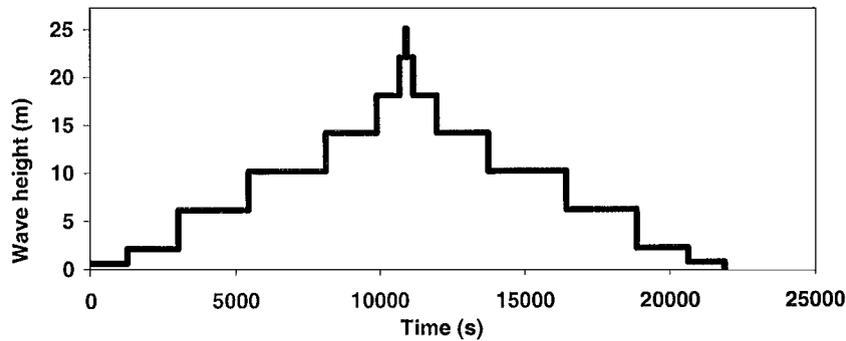


Figure 5. Sequence for the design waves (series 2).

Table III. Calculation of the number of cycles for series 2.

Wave order	Wave height (m)	No. of waves in a storm	Value of N
1	0.6	249	From 1 to 249.
2	2.1	245	Wave height 0–0.6 m: from 250 to 494 Wave height 0.6–2.1 m: from 1 to 245
3	6.1	243	Wave height 0–0.6 m: from 495 to 737 Wave height 0.6–2.1 m: from 246 to 488 Wave height 2.1–6.1 m: from 1 to 243
4	10.1	236	Wave height 0–0.6 m: from 738 to 973 Wave height 0.6–2.1 m: from 489 to 724 Wave height 2.1–6.1 m: from 244 to 479 Wave height 6.1–10.1 m: from 1 to 236
5	14.1	141	Wave height 0–0.6 m: from 974 to 1115 Wave height 0.6–2.1 m: from 725 to 865 Wave height 2.1–6.1 m: from 480 to 620 Wave height 6.1–10.1 m: from 237 to 377 Wave height 10.1–14.1 m: from 1 to 141

Suppose all stress increments caused by all waves are included inside the existing yield surface (i.e. virgin yielding will not occur for the loading caused by the entire group of waves), and the value of N for the first wave is equal to 1. The cycle numbers calculated according to the proposed method for this sequence are as listed in Table III. For this case the more general Equation (6) is required for the calculation of pore pressure development. Since N is defined, δu can be computed.

A numerical example can be found from a report by Liu and Carter [12] to illustrate the use of the equations derived in this section to calculate the development of pore pressure of a soil element under cyclic loading.

Similarly, the empirical equations describing the accumulation of pore pressure for repeated loading, such as those proposed by Bjerrum [6] and Tsatsanifos and Sarma [7], can also be extended in the same manner for repeated loading along general stress paths.

3.2. Soil deformation under repeated loading with a large number of cycles

To describe the deformation of soil under repeated loadings with a large number of cycles, say more than 20 cycles, it is frequently necessary to include the number of cycles N as a parameter in constitutive models. This type of approach is clearly expedient and pragmatic, but for engineering application it is generally preferable to the alternative more rigorous approach [2,4,5]. The alternative method, which follows the strict requirements of the theory of elastoplasticity, is too complicated and also often yields less accurate numerical solutions. An example is given here to illustrate this point.

In the design and rehabilitation of highway and airport pavements, the empirical equation proposed by Lentz and Baladi [13] is often used to estimate the permanent deformation of the pavement [4]. The equation is as follows:

$$\varepsilon_p = a + b \ln N \quad (9)$$

where ε_p is the accumulated permanent strain, a and b are two empirical constants and their values are dependent on the materials tested and the stress paths applied and N is the number of cycles of repeated loading.

The values of a and b are usually determined from conventional drained triaxial tests with fixed confining pressure. Consequently, for a given material, a and b are dependent on the amplitude of the axial stress and confining stress. The values for a and b identified for a medium dense North Michigan sand are listed in Table IV. In this table, σ'_d and σ'_r are the amplitude of the deviator stress (the maximum difference between the axial stress and the confining pressure) and the confining stress, respectively.

Empirical equation (9) should be applicable to calculating the permanent strain for cyclic loading along any stress path, now that the cycle number N for repeated loading along general stress paths can be determined via the proposed method. To illustrate the significance of the proposed method, a simple hypothetical example is given here. Suppose a cyclic test is carried out on a soil identical to that used by Lentz and Baladi [13]. During the test the confining stress is kept constant with $\sigma'_r = 35$ kPa. The sample is firstly subjected to 1 million cycles of uniform repeated loading with $\sigma'_d = 129$ kPa, followed by 3 million cycles of uniform repeated loading with $\sigma'_d = 119$ kPa, followed by 10 million cycles of uniform repeated loading with $\sigma'_d = 101$ kPa, followed by 100 million cycles of uniform repeated loading with $\sigma'_d = 69$ kPa, and finally followed by 1000 million cycles of uniform repeated loading with $\sigma'_d = 34$ kPa. Based on the available information, the key question to answer is how much permanent strain is accumulated by this sample.

According to equation (9), the permanent axial strain for cyclic loading along the same stress path from cycle N_0 to cycle N_1 can be calculated by

$$\Delta\varepsilon_p |_{N_0 \text{ to } N_1} = b \ln \frac{N_1}{N_0} \quad (10)$$

Table IV. Values of a and b for a medium dense North Michigan sand (after Lentz and Baladi [12]).

Test	$\sigma'_r = 35$ kPa $\sigma'_d = 34$ kPa	$\sigma'_r = 35$ kPa $\sigma'_d = 69$ kPa	$\sigma'_r = 35$ kPa $\sigma'_d = 101$ kPa	$\sigma'_r = 35$ kPa $\sigma'_d = 119$ kPa	$\sigma'_r = 35$ kPa $\sigma'_d = 129$ kPa
a	9.9×10^{-6}	2.967×10^{-4}	8.287×10^{-4}	8.685×10^{-4}	1.2331×10^{-3}
b	1.935×10^{-5}	6.912×10^{-5}	1.6096×10^{-4}	2.283×10^{-4}	4.0374×10^{-4}

Table V. Calculation of permanent strains.

Stress path $\sigma'_r = 35$ kPa	Total no. of cycles	Values of N for every cycle	$\Delta\varepsilon_p$ for each stress path	ε_p if each stress path is applied to a fresh sample
$\sigma'_d = 129$ kPa	1×10^6	From 1 to 1×10^6	0.6811%	0.6811% ($N = 1 \times 10^6$)
$\sigma'_d = 119$ kPa	3×10^6	From ($1 \times 10^6 + 1$) to 4×10^6	0.0316%	0.4273% ($N = 3 \times 10^6$)
$\sigma'_d = 101$ kPa	1×10^7	From ($4 \times 10^6 + 1$) to 1.4×10^7	0.0202%	0.3423% ($N = 1 \times 10^7$)
$\sigma'_d = 69$ kPa	1×10^8	From ($1.4 \times 10^7 + 1$) to 1.14×10^8	0.0145%	0.157% ($N = 1 \times 10^8$)
$\sigma'_d = 34$ kPa	1×10^9	From ($1.14 \times 10^8 + 1$) to 1.114×10^9	0.0044%	0.0411% ($N = 1 \times 10^9$)

There are five different stress paths for this repeated loading, and the previous loading history has an influence on the value of N appropriate to subsequent loading. The permanent axial strain accumulated under the full pattern of repeated loading is calculated to be 0.7518%.

The details of these calculations are shown in Table V. For comparison, the permanent axial strains produced if each of the five parcels of the cyclic loading is applied to a fresh identical sample have also been calculated.

4. CONCLUSION

A new method for defining the number of cycles, N , during repeated loadings has been presented. It is proposed that the value of N is associated with the variation of the pseudo-yield surface. A stress increment which causes the pseudo-yield surface to expand is defined as loading, and a stress increment which causes the pseudo-yield surface to shrink is defined as unloading. The cycle number N for reloading and unloading is calculated separately. Based on the proposed method, a large number of valuable semi-empirical formulae and some constitutive equations which contain the number of cycles N as a parameter can thus be applied rationally for engineering calculation of soil deformation under repeated loadings that may be either simple or complicated in nature.

It should be pointed out that the method to calculate the cycle number N proposed in this paper is rational, but not unique. Other suggestions may also be possible. The validation of the proposed method for use with the vast number of equations using N as a variable for cyclic loading along complicated stress paths still requires checking. Such validations are beyond the scope of this paper, and have not yet been attempted because of the lack of necessary experimental data. However, a possible means to check the validity experimentally is suggested as follows.

- (a) Determine the response of samples of a given soil to uniform repeated loadings (with N being constant in each cycle). Several sets of different cyclic stress paths should be carried out.

- (b) Based on the experimental data, determine the parameters for the empirical equation describing the soil response under uniform cyclic loading.
- (c) Design and conduct an experiment with cyclic loading along a complicated stress path.
- (d) Predict the strain or appropriate response quantity for the designed test according to the empirical equation, the measured equation parameters, and the cycle number determined by the proposed method.
- (e) Compare the prediction with the measured data.

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