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Constitutive model of unsaturated structured soils under cyclic loading

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ABSTRACT: Unsaturated structured soil has been one of research focuses in geotechnical mechanics. This paper aims to investigate the influence of structure degradation and suction variation on unsaturated soils subjected to cyclic loading. Within the framework of plastic bounding surface model, the structure damage theory and unsaturated soil mechanics, an elasto-plastic model for unsaturated soils under cyclic loading has been elaborated. Firstly, on the basis of Barcelona Basic Model, the water retention curve is employed to unite the suction with mechanical stresses under constant water condition. Secondly, the size of yielding surface is assumed to depend on the soil structure damage, and a law of structure degradation is given, linking the structure damage with accumulated plastic strain increments. Besides, by incorporating the concept of mobile mapping origin, the bounding surface model is modified to unify the mapping rule used during both loading and unloading, to simulate the phenomenon of hysteretic properties under cyclic loading. Comparisons between experimental results or published data and model simulations show that the elaborated model is capable of describing unsaturated soil behavior under cyclic loading.

1 INTRODUCTION

The long-term deformation of foundations in the transportation system like the subway, expressway, high speed railway, has been one of the very important researches in the international geotechnical engineering. In China, the long-term performance of the metro has caused great differential settlements along the subway tunnel in Shanghai, with the maximum value over 10 cm (Ye Zhu & Wang, 2007). In Japan, the settlement in the embankment of one expressway has been accumulated up to 1~2 m during its initial 5 years' operation (Yasuhara, Hirao & Hyodo, 1988). And in France, instability problems like the formation of sinkholes have been observed near the Northern French high speed railway, with maximum depth up to 1 m (Cui & Terpereau, 2004). Development of similar foundation differential settlements in transportation system will lead to serious safety problems and huge economic lost.

With further in-situ measurements and laboratory experiments, some soils are probably within the unsaturated state for the sake of climate, ventilation, and the component of soil itself. Therefore, it's important to further study the behaviors of unsaturated cemented soils subjected to cyclic transportation loads.

Since most current constitutive models of soils are developed with respect to remolded soils, the phenomena of cementation damage in natural soils can not be described reasonably. It is necessary to establish some constitutive models which are capable of modeling the cementation damage of soils (Shen, 1996). Generally, the soil structure is attributed to the specific arrangement of solid particles and the cementation of sediments from the pore water between the interfaces of solid particles. Damage of cementation in structured soils under applied external loads is not only able to change the size and shape of initial yielding surface, but also to decrease the whole stiffness of soils (Burghignoli, Miliziano & Soccodato, 1998, Sharma & Fahey, 2003a&b). Once the cementation is damaged completely, the original structured soil approaches to the similar critical state with the correspondingly unstructured soils (Gens & Nova, 1993, Chai, Cui & Lu, 2005). Vaunat and Gens (2003) assumed that soils are composed of solid particles, bond, and void. The bond is taken as a brittle material, whose damage process symbolizes the deterioration of soil structure, and then the Modified Cam-Clay Model is incorporated to describe the constitutive behavior of structured soils. On the other side, Rouainia and Muir Wood (2002), Wei and Huang (2007) modified the function of yielding surface to cover the structural behavior of soils, and attributed the cementation damage to the development of plastic deformation of soils. Bounding surface model and dynamic hardening laws are employed to build up their both constitutive models on the basis of MCCM. It can be seen that the first method above (like Vaunat and Gens) is presented from the microscopic view to describe the cementation damage, but the second one is developed from the macroscopic view, both of which are representative of current researches on structured soils.

Under cyclic loading, the deformation of soils is obviously non-linear and usually hysteretic in terms of strain vs. stress. Since the classical elasto-plastic theory assumes a complete elasticity within the yielding surface, it's unable to account for the non-linearity of modulus, change of pore pressure, the incurrence of hysteretic circles, the accumulation of plastic strains, and other characteristic behaviors under cyclic loading (Pastor, Zienkiewicz & Leung, 1985, Pastor, Zienkiewicz & Chan, 1985, Krieg, 1975). The multi-yielding surface theory on the basis of plastic hardening modulus provides a general framework to describe soil behaviors under cyclic loadings. However the multi-yielding surface model is somewhat complicated due to both the necessity to tracking all yielding surfaces in the stress space when modeling and the many parameters difficult to be determined from experiments. Simplifications are made by Krieg (1975), and Dafalias and Popov (1975), thus leading to the current bounding surface model. In this simplified model, the bounding surface and loading surface are defined, and the yielding surface between these two limiting surfaces is determined by interpolation functions. Also, the plastic modulus on the loading surface is a combined function of the distance between the current stress point and the conjugated stress point on the bounding surface. Furthermore, Dafalias and Herrmann (1982) developed the single yielding surface model by degrading the mobile loading surface into one point. As one of the effective tools in modeling soil behaviors under cyclic loading, the bounding surface model has been applied in various kinds of soils such as clays and sands (Pastor, Zienkiewicz & Chan, 1985, Liang & Ma, 1992, Li & Huang, 2007).

Chai, Cui & Lu (2005) employed the bounding surface model and generalized plastic model to predict the softening and liquidation phenomena of French loess. On this basis, Yang, Cui, Pereira & Huang (2008) further established a constitutive model for unsaturated cemented soils under cyclic loading, which describes the structure damage from the microscopic view, but which seems to be complicated in formulation and not easily applicable to engineering. Here this paper aims to simplify and improve the previous model (Yang, Cui, Pereira & Huang, 2008) from the macroscopic view so that to cover more effectively the behavior of unsaturated structured soils under cyclic loading.

2 FORMATION OF THE CONSTITUTIVE MODEL

2.1 *Description of unsaturated behaviors of soils*

Experiments show that the increase in suction can enhance the strength and stiffness of unsaturated soils (Alonso, Gens & Josa, 1990, Wheeler & Sivakumar, 1995). Alonso, Gens & Josa (1990) established the first critical state unsaturated soil constitutive model with two stress state variables, net stress, which is the difference of total stress and pore air pressure, and suction, which is the difference of pore air pressure and pore water pressure. This model is named after Barcelona Basic Model (BBM), and favorably accepted in the unsaturated soil mechanics.

This paper employs the Loading-Collapse yielding surface adopted in BBM, and incorporates the van Genuchten Soil Water Retention Curve (van Genuchten, 1980) to combine the suction together with external stresses to describe the hydro-mechanical behavior of unsaturated soils.

(1) LC yielding surface

$$\frac{p_0}{p^c} = \left(\frac{p_0^*}{p^c}\right)^{\frac{\lambda_m(0)-\kappa_m}{\lambda_m(s)-\kappa_m}} \tag{1}$$

where p_0^* and p_0 are the preconsolidation stresses in saturated and unsaturated state respectively, p^c is the reference stress, $\lambda_m(s)$ is the deformation coefficient in unsaturated state, defined as

$$\lambda_m(s) = \lambda_m(0)\left[(1-r_s)\exp(-\beta_s s) + r_s\right] \tag{2}$$

where, r_s is a constant related to the maximum stiffness of soils, β_s is a constant controlling the rate of the change of soil stiffness with suction increase, $\lambda_m(0)$ is the deformation coefficient of soils in saturated state.

(2) Soil water retention curve

$$S_r(s) = \left(\frac{1}{1+(\alpha \cdot s)^n}\right)^m \tag{3}$$

where α, n and m are fitting parameters from experiments.

2.2 *Mechanical description of soil structure*

To describe the damage of soil structure from a macroscopic view, it generally begins with the constitutive models of totally remolded soils like MCCM, and further to fabricate the enlarged yielding surface due to the existence of initial structure of soils. Subsequently, the evolution of the previous defined yielding surface is constructed to model the damage process of the natural soils (Rouainia and Muir Wood, 2002).

This paper assumes the structure damage of soils only influences the size but not the shape of yielding surface. During loading, the structure of soils degrades gradually, and thus causes the yielding surface of the structured soils to contract towards that of totally remolded soils.

Define a damage factor r_d as

$$r_d = \bar{p}_0 / p_0 \tag{4}$$

where $\bar{p}_0$ and p_0 are the yielding stresses in structured and unstructured soils separately.

And assume the damage factor, r_d to be the exponential function of plastic damage strain ε_d^p, and can be expressed as

$$r_d = 1 + (r_{d0} - 1)\exp(-k_d \varepsilon_d^p) \tag{5}$$

where r_{d0} is the initial structural factor, k_d is the exponent describing the degrading rate of structure with plastic deformation, and the plastic damage strain is given as the function of absolute accumulation of plastic strains, as

$$\varepsilon_d^p = \int \sqrt{(1-A)\cdot(d\varepsilon_p^p)^2 + A\cdot(d\varepsilon_q^p)^2} \tag{6}$$

where $d\varepsilon_p^p$ is the rate of plastic volumetric strain, $d\varepsilon_q^p$ is the rate of plastic deviatoric strain, and A is a parameter expressing the proportion of the influence of $d\varepsilon_p^p$ and $d\varepsilon_q^p$ on the structure damage of soils.

Therefore, the constitutive model of unsaturated structured soils can be depicted in Figure 1 in *p-q-s* stress space. p_{bc} and p_s are the components of tension strength of soils due to the soil structure and suction effects separately.

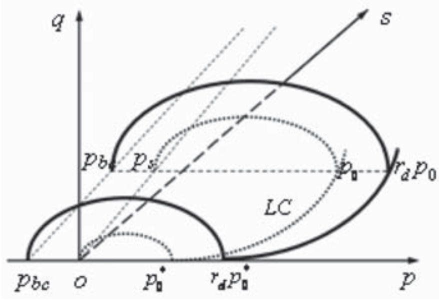


Figure 1. Three-dimensional view of yield surfaces in (p, q, s) stress space.

2.3 *Bounding surface constitutive model*

Within the framework of bounding surface model and critical state theory, one elasto-plastic constitutive model for unsaturated structured soils can be obtained, which not only considers the collapse behavior and soil-water retention curve of unsaturated soils, but incorporates the structure degradation. Formulation of the model is specified as follows.

(1) Choice of stress variables

Regarding the structure of soils, the damage stress variables, $\sigma = (\bar{p}, \bar{q})^T$ are given as

$$\bar{p} = p + \bar{p}_{bc}; \quad \bar{q} = q \tag{7}$$

Notes: in this paper the net stress, deviatoric stress, and suction are denoted as p, q, and s, respectively.

(2) Bounding surface equation

According to the critical state theory, the bounding surface equation can be written as

$$F(\boldsymbol{\sigma}, s, p_0^*, \varepsilon_d^p) = \bar{q}^2 - M^2(\bar{p} + p_s)(\bar{p}_0 - \bar{p}) \tag{8}$$

where M is the slope of the critical state line.

(3) Hardening law

During the plastic loading, the hardening parameter p_0 can be given by the plastic volumetric strain, and expressed as

$$dp_0 = p_0 \cdot \frac{1 + e_m}{\lambda_m(s) - \kappa_m} \cdot d\varepsilon_{pm}^p \tag{9}$$

where e_m is the void ratio, λ_m, κ_m are the slopes of normal consolidation line and unloading-reloading line.

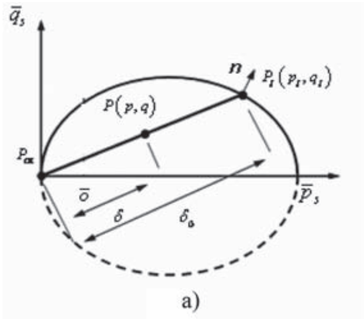
Furthermore, the associated flow rule is adopted. With the consistency condition, the flow rule, and the hardening law, the plastic modulus H_L^{BS} on the yielding surface during loading or unloading can be deduced.

(4) Imaging rule and mobile mapping center

From the bounding surface theory (Dafalias and Herrmann, 1982), the traditional linear mapping rule is adopted in this paper. The plastic modulus H_L at the current stress point is proportional to the corresponding modulus at the conjugate stress point on the bounding surface, thus the plastic deformation within the bounding surface described.

Considering the incorporation of unsaturated aspect and structure degradation behaviour, in this paper the mapping rule is defined in the stress space of $(\bar{p}_s, \bar{q}_s)$ (See Fig. 2).

The mapping rule in the traditional bounding surface model is modified to simulate the hysteretic circles during the unloading-reloading process.



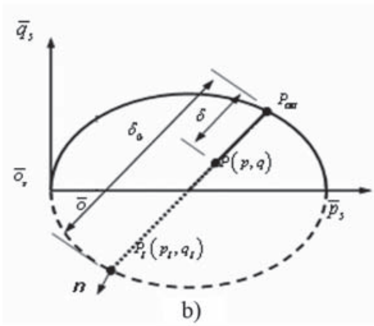


Figure 2. Mapping rule in bounding surface model.

According to Masing, the unloading can be considered as the inverse loading with respect to the traditional loading. Therefore, both loading and unloading can be unified into one general loading, further leading the concept of mobile mapping center in the bounding surface model. To be specific, during traditional loading, the mapping centre can be taken as the left interaction point P_{OL} of the yielding surface and the abscissa $\bar{p}_s$ (See Fig. 2a). And during the inverse loading (unloading), the mapping center moves correspondingly to the beginning point of the inverse loading (See Fig. 2b).

Therefore, the specific mapping rule can be given as follows

$$H_{L/U} = H_{L/U}^{BS}\left(\frac{\delta_0}{\delta}\right)^{r_{L/U}} \tag{10}$$

where $r_{L/U}$ is the mapping exponent in unified loading, and varies with the structure degradation as follows.

$$r_{L/U} = r_d \cdot r_{L/U0} \tag{11}$$

where $r_{L/U0}$ is the mapping exponent of equivalent remolded soils, δ_0 and δ are the distances from the imaging point on the bounding surface and the current stress point to the imaging center, listed as

$$\delta_0 = \sqrt{\left(p_I - p_{OL/U}\right)^2 + \left(q_I - q_{OL/U}\right)^2} \tag{12}$$

$$\delta = \sqrt{\left(p - p_{OL/U}\right)^2 + \left(q - q_{OL/U}\right)^2} \tag{13}$$

Finally, the rate of plastic strains $d\varepsilon^p$ during the general loading can be gained according to the flowing rule.

2.4 *Determination of model parameters*

In this model, most parameters can be obtained from common experiments. But some other parameters about the structure degradation like r_{d0}, k_d, A and bounding surface model like $r_{L/U0}$ can't be determined easily. In the following simulations these parameters are determined by parameter optimization method.

3 MODEL VALIDATIONS

In order to testify the efficiency of the model established above, two samples under the simple shearing and the cyclic triaxial shearing with constant water content are simulated separately with the model proposed above, and results are compared with the laboratory experiments and relevant data in previous documents.

3.1 *Simple shearing test under constant water content*

With reference to experimental data in Alonso, Gens & Josa (1990), the sample is sheared under constant net stress, $p = 150\ kPa$, and constant water content. Although tests in Alonso et al. (1990) are simulated at constant net stress and constant suction ($s = 100, 200, 300\ kPa$), their data are still very useful for the validation of this model. Values of parameters are listed in Table 1. And the comparison between the simulation of this paper and the know data is depicted in Figure 3.

It can be seen that with the cooperation of external applied stress, suction and degree of saturation, the calculated values of suction and soil strength decreases with soil compression during shearing, in accordance with conclusions among others (Alonso, Gens & Josa, 1990, Cui & Delage, 1993).

Meanwhile, when compared with totally remolded soils, the strength of structured soils is bigger, and its decreasing range of suction is smaller.

Table 1. Parameters used in simulation of the shear test.

p_0* (kPa)	e_0	u_m	$\lambda_m(0)$	κ_m	κ_s	M
150	0.90	0.35	0.20	0.02	8e-3	1.0
p^c (kPa)	r_{L0}	k_s	r_s	n	m	β_{Sr}
100	1.4	0.60	0.75	0.5	3	0.01
s_0 (kPa)	α	r_{d0}	k_d	A		p_{bc} (kPa)
300	1.25e-2	1.35	3.5	0.25		10

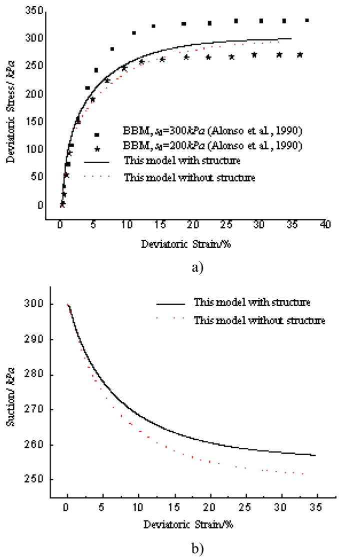


Figure 3. Response of model to shear tests with constant water content and constant mean net pressure.

But the structured soil finally tends to the same critical state as the totally remolded soils. This result also can be found in papers of Leroueil & Vaughan (1990), Gens & Nova (1993), and Chai, Cui & Lu (2005).

3.2 *Cyclic triaxial shearing tests of unsaturated French loess under constant water*

French loess is known as the typical unsaturated cemented soils (Cui & Terpereau, 2004). Here the initial water content of studied soils is 23%. The sample is first consolidated with 25 *kPa* confined stress, and sheared with single or multiple levels of cyclic loading. Values of parameters are listed in Table 2.

First one single level of cyclic shearing between 0 and 75 *kPa* has been simulated with this model. Results in terms of deviatoric stress vs. deviatoric strain, and axial strain vs. cyclic loading number are shown in Figure 4. It seems that this model is capable to offer fairly nice predictions on the strain accumulation and also the hysteresis under cyclic loading.

To further validate the efficiency of this model in predicting the plastic strain accumulation under cyclic loading, multiple levels of cyclic loading is carried out. The initial deviatoric stress varies between 0 and 15 *kPa*, afterwards the peak value of the deviatoric stress in each level is increased by 15 *kPa*. In each stress level, the cycling of 100 times is applied till the sample is destroyed. Results in Figurwe 5 shows that the model proposed in this paper can predict the deformation of soils under cyclic loading.

Table 2. Parameters used in simulation of the cyclic triaxial shear tests.

p_0* (kPa)	e_0	u_m	$\lambda_m(0)$	κ_s	M	k_s
700	0.93	0.25	0.17	0.01	1.1	0.02
p^c (kPa)	κ_m	r_{L0}	r_{U0}	β_s	r_s	β_{Sr}
25	0.012	1.36	1.85	0.01	0.75	0.16
s_0 (kPa)	p_{bc} (kPa)	r_{d0}	k_d	A	n	m
1000	10	1.35	3.5	0.25	5.75	0.06

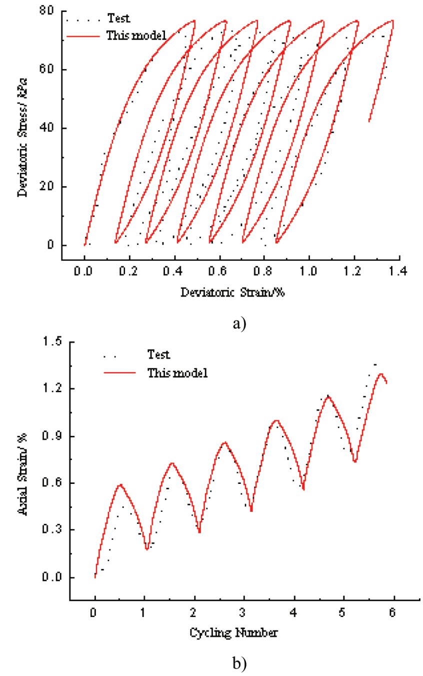


Figure 4. Response of model to single level cyclic triaxial shear tests with constant water content.

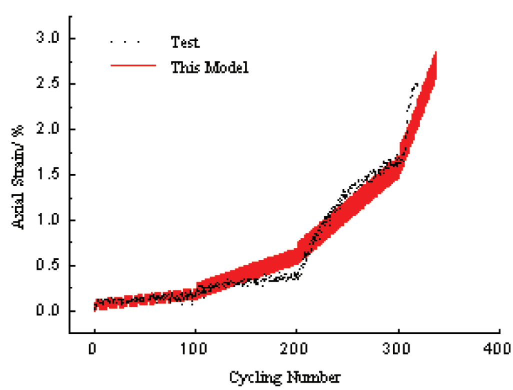


Figure 5. Axial strains of samples with constant water content under multi-level cyclic loading.

4 CONCLUSION

The mechanical behavior of unsaturated structured soils under cyclic loading has been studied in this paper. Within the framework of bounding surface model, one elasto-plastic model is proposed with the combination of structure damage theory and unsaturated soil mechanics. Meanwhile, the mobile mapping origin is introduced to modify the traditional mapping rule in bounding surface model, unifying the loading and unloading into one generalized loading. Comparisons between model predictions and experimental results or know data show that this model is capable to simulate the mechanical behavior of unsaturated structured soils mentioned in this paper both qualitatively and quantitatively well. However, the influence of deformation on the soil water retention curve is not included in this paper. And some parameters are difficult to be measured directly. Therefore, more efforts are required for the improvement and validation of this model.

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